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LAGEOS I once-per-revolution force due to solar heating

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Abstract. Photon thrust from the solar heating of the LAGEOS I satellite appears to explain much of the eccentricity variations seen in the satellite's orbital elements. We invoke a thermal model of LAGEOS I in which the photon thrust from solar heating is directed along the satellite's spin axis and functionally depends only on the cosine of the angle between the Sun's position and the spin axis. We calibrated the amplitude of the force from the 1980-1983 equivalent along-track acceleration derived from the observed orbital perturbations; during this time the spin axis position is assumed to be known and to be that at orbit injection. The photon thrust from this simple thermal model, plus later spin axis positions obtained from Sun glint data (which show LAGEOS I to be precessing), give reasonable agreement with the observed along-track acceleration in the time period 1988-1995. Thus much of the eccentricity variations seem to be due to thermal thrust and do not have a geophysical origin (atmospheric tides) as has been proposed. However, our solar heating model does not appear to explain the highest peaks and deepest troughs seen in the along-track acceleration, indicating the need for a better thermal model and consideration of other forces, such as that due to anisotropic reflection.

Introduction

The LAGEOS satellite is in a near circular orbit about the Earth at an altitude of approximately 5900 km and with an orbital inclination of about 110°. LAGEOS was launched in 1976 to investigate numerous geophysical phenomena, such as plate motion, polar motion, tides, etc. [*Cohen and Smith*, 1985]. This satellite is now designated as LAGEOS I because it was joined by a sister satellite in 1992, LAGEOS II.

The LAGEOS I satellite is experiencing unknown forces which have magnitudes of about 1/20 that of direct solar radiation pressure and a frequency of one cycle per revolution of the satellite about the Earth. These forces reveal themselves in long-period excitations in LAGEOS's orbital eccentricity [e.g., *Tapley et al.*, 1993]. These forces are most easily modeled by assuming they are all along-track, because the other orthogonal components, if they exist, are too highly correlated with the along-track acceleration to be determined independently. However, these other components certainly exist in the mechanism proposed here. Hence by ignoring these other components, we are assuming here what might be called the "equivalent along-track" force.

In the following we investigate whether the observed perturbations could be due to photon thrust from solar heating of the satellite, as independently proposed by V. J. Slabinski (private communication, 1994), J. Ries (private communication, 1994), and others [e.g., *Farinella and Vokrouhlicky*, 1996]. The idea is as follows: the spin axis of the satellite is tipped toward the Sun, so that one hemisphere of the satellite is preferentially heated over the other, as shown in Figure 1. Due to the rapid rotation of the satellite, to a first approximation the longitudinal variation of temperature around the satellite is averaged out, leaving only a latitudinal variation of temperature (latitude and longitude referring here to coordinates fixed in the body of the satellite). Since photons have momentum, the hotter hemisphere carries away more momentum than the colder, so that LAGEOS is subject to thermal thrust. By the symmetry of the temperature distribution, the net force on LAGEOS will be along the spin axis of the satellite, in the direction away from the hotter hemisphere. This force will have an along-track component, with a frequency of once-per-revolution, just like the observed forces. It will also have radial and cross-track components.

The amplitude of the along-track component will change with time due to the varying Sun-orbit-spin axis geometry. Not only does the orbit precess, but so does the spin axis of LAGEOS. The precession of the spin axis is mainly due to torques arising from electrical currents in the metallic satellite as it spins in the Earth's magnetic field. There is also a smaller gravitational torque: the outer aluminum surface of LAGEOS is spherical, but inside it has a cylindrical beryllium-copper core, giving the satellite an oblate mass distribution. The Earth's gravitational field acts on this mass distribution, causing a torque on LAGEOS.

Several theoretical models and computer programs have been developed to predict the precession of LAGEOS [*Bertotti and Iess*, 1991; *Habib et al.*, 1994; *Farinella et al.*, 1996). However, these theories have not yet provided an acceptable model, perhaps due to the complexity of the satellite in

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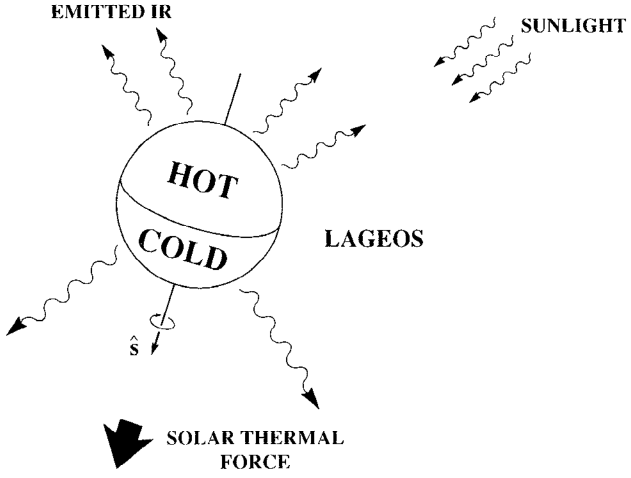

Figure 1. Schematic of solar heating when LAGEOS is rapidly rotating. The thermal force is along the satellite's spin axis and away from the hotter hemisphere.

comparison to the simple structures that can reasonably be implemented. For this reason, we must resort to the somewhat sporadic direct observations of the orientation of the spin axis. The observational procedure is to detect and time the reflections of the Sun from the front faces of the laser retroreflectors on LAGEOS's surface. Using the timing information of the Fresnel reflections from a single row of retroreflectors, and the known observatory-satellite-Sun geometry, the spin axis orientation can be determined. The required angular information may be obtained from the pointing of the telescope or the satellite's orbital elements. This observational and analytical procedure results in a determination of the orientation of the spin axis in most cases to an accuracy of a few degrees [*Currie*, 1994], leading to errors in the acceleration of at most a few picometers per second squared.

Using the orientation of the spin axis and the thermal model to predict the photon thrust due to solar heating, we find that it has the same signature and explains much of the along-track acceleration obtained from satellite laser ranging (SLR) data. Thus the observed forces may not have a geophysical origin, such as the S_1 atmospheric tide, as proposed by *Nerem et al.* [1994]. However, our model apparently cannot account for the highest peaks and deepest troughs in the along-track acceleration, so that a more sophisticated model may be needed or perhaps some part of the signal is in fact due to tides. Other forces may need to be considered as well, such as the anisotropic reflection force [*Rubincam et al.*, 1987, p. 11,667; *Scharroo et al.*, 1991; *Sengoku et al.*, 1995]. This force results from a latitudinal asymmetry in the reflectivity or specularity of the satellite, so that if the Sun stands on the satellite's equator, for example, LAGEOS will still feel a net force along the spin axis. We do not consider the anisotropic reflection force here, nor do we deal with solar radiation pressure. Solar radiation pressure will have a signature identical with the thermal force (J. C. Ries, private communication, 1996); thus any errors in the average reflectivity of LAGEOS I will be absorbed into our model. Also, we deal only with LAGEOS I and not LAGEOS II. For a review of other thermal forces, see *Rubincam* [1990a] and *Ries et al.* [1993].

Theoretical Expression

Let (x, y, z) be an inertial coordinate system centered on the Earth, as shown in Figure 2, where the x axis points to the vernal equinox, the z axis points along the rotation axis of the Earth, and the y axis makes the system right handed. Let the unit spin vector $\hat{s}$ have the components

$$\hat{\mathbf{s}} = s_x\,\hat{\mathbf{x}} + s_y\,\hat{\mathbf{y}} + s_z\,\hat{\mathbf{z}}$$

where $\hat{x}, \hat{y}, \hat{z}$ are unit vectors along their respective axes. The unit vector $\hat{t}$ tangent to the orbit is

$$\begin{aligned}\hat{\mathbf{t}} = {} & [-\cos\Omega\sin(\omega+f) - \cos I\sin\Omega\cos(\omega+f)]\,\hat{\mathbf{x}} \\ & + [-\sin\Omega\sin(\omega+f) + \cos I\cos\Omega\cos(\omega+f)]\,\hat{\mathbf{y}} \\ & + [\sin I\cos(\omega+f)]\,\hat{\mathbf{z}}\end{aligned}$$

for a circular orbit, where ω is the argument of perigee, f is the true anomaly, I is the inclination, and Ω is the nodal position for LAGEOS. The unit vector $\hat{r}_s$ pointing from the Earth to the Sun is

$$\begin{aligned}\hat{\mathbf{r}}_s = {} & [\cos(\omega_s + f_s)]\,\hat{\mathbf{x}} \\ & + [\cos I_s\sin(\omega_s + f_s)]\,\hat{\mathbf{y}} \\ & + [\sin I_s\sin(\omega_s + f_s)]\,\hat{\mathbf{z}}\end{aligned}$$

where the subscript s refers to the Sun's orbital elements in the (x, y, z) system. In this system $I_s = 23.5°$.

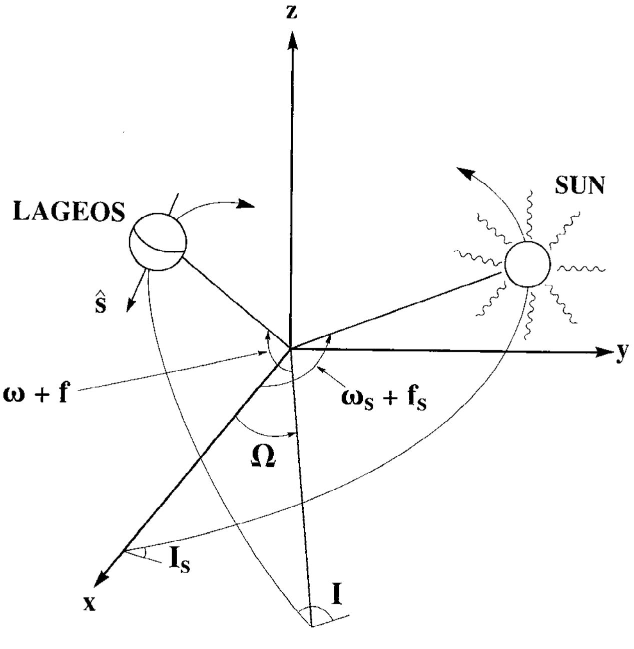

Figure 2. The geometry of the orbit, Sun, and spin axis. The x axis points in the direction of the vernal equinox, while the z axis points along the Earth's north pole.

The once-per-revolution force is assumed to have the form

$$\ddot{\mathbf{r}} = -b\,(\hat{\mathbf{s}} \cdot \hat{\mathbf{r}}_S)\,\hat{\mathbf{s}} \tag{1}$$

where $\ddot{r}$ is the acceleration of LAGEOS, and b is a positive constant. The factor $(\hat{s} \cdot \hat{r}_S)$ represents the solar heating effect as it affects the orbital acceleration; the heating thus has a cosine dependence. V. J. Slabinski (A numerical solution for LAGEOS thermal thrust: The rapid spin case, submitted to *Celestial Mechanics*, 1995, hereafter referred to as V. J. Slabinski (submitted manuscript, 1995)) has a much more sophisticated thermal model than that presented here but finds the dominant term to be of the form given by (1); the next largest term, which depends on the cube of the cosine, is about 1% the dominant term. Hence we will use the above simple expression in this paper.

The along-track component of this acceleration will be

$$(\ddot{\mathbf{r}} \cdot \hat{\mathbf{t}}) = -b\,(\hat{\mathbf{s}} \cdot \hat{\mathbf{r}}_S)\,(\hat{\mathbf{s}} \cdot \hat{\mathbf{t}}). \tag{2}$$

This can be written

$$(\ddot{\mathbf{r}} \cdot \hat{\mathbf{t}}) = A \sin(\omega + f) + B \cos(\omega + f). \tag{3}$$

Since $\omega + f$ circulates once per revolution of LAGEOS, the above expression has the required form.

The coefficients A and B are

$$\begin{aligned} A = \frac{b}{2}\{ & [\, s_x s_y (1 + \cos I_s) + s_x s_z \sin I_s] \sin(\Omega + \omega_s + f_s) \\ & + [\, s_x^2 - s_y (s_y \cos I_s + s_z \sin I_s)\,] \cos(\Omega + \omega_s + f_s) \\ & + [\, s_x s_y (1 - \cos I_s) - s_x s_z \sin I_s\,] \sin(\Omega - \omega_s - f_s) \\ & + [\, s_x^2 + s_y (s_y \cos I_s + s_z \sin I_s)] \cos(\Omega - \omega_s - f_s) \,\} \end{aligned} \tag{4}$$

$$\begin{aligned} B = \frac{-b}{2}\{ & \\ & [\, -s_x^2 \cos I + s_y \cos I\,(s_y \cos I_s + s_z \sin I_s)] \sin(\Omega + \omega_s + f_s) \\ + & [\, s_x s_y \cos I\,(1 + \cos I_s) + s_x s_z \cos I \sin I_s\,] \cos(\Omega + \omega_s + f_s) \\ - & [\, s_x^2 \cos I + s_y \cos I\,(s_y \cos I_s + s_z \sin I_s)\,] \sin(\Omega - \omega_s - f_s) \\ + & [\, s_x s_y \cos I\,(1 - \cos I_s) - s_x s_z \cos I \sin I_s\,] \cos(\Omega - \omega_s - f_s) \\ + & [\, 2 s_z \sin I\,(s_y \cos I_s + s_z \sin I_s)\,] \sin(\omega_s + f_s) \\ + & [\, 2 s_x s_z \sin I\,] \cos(\omega_s + f_s) \,\} \end{aligned} \tag{5}$$

In these equations, $s_x = \sin\theta \cos\lambda$, $s_y = \sin\theta \sin\lambda$, and $s_z = \cos\theta$, where θ is the colatitude of the LAGEOS spin axis measured from the Earth's north pole, and λ is the east longitude of the LAGEOS spin axis measured from the vernal equinox. The A and B coefficients given by (4) and (5) will slowly change with time as the orbit and spin axis precess and the solar position varies. Expressions similar to (3)-(5) can be written for the radial and orbit-normal accelerations, but these will not be given here.

Figure 3 shows the amplitude of the sine part of the once-per-revolution force (corresponding to A in (4) above). We do not use the sine because this part of the force is generally less than half as large as the cosine part, so that it is more obscured by noise and perhaps unmodeled forces. Figure 4 shows the cosine part of the once-per-revolution force (corresponding to B in (5) above) for LAGEOS I as recovered from the SLR data, after using the GEODYN orbit determination program to take out other known forces, such as gravitational forces and solar radiation pressure. We rely only on the cosine part (Figure 4) in the analysis given below.

We take into account the shadowing of LAGEOS I as it enters and exits the Earth's umbra by decreasing the forces in proportion to the time the satellite spends in the shadow. We ignore the anisotropic reflection force proposed by *Rubincam et al.* [1987, p. 11,667) and W. M. Kaula (private communication, 1987) and investigated by *Scharroo et al.*

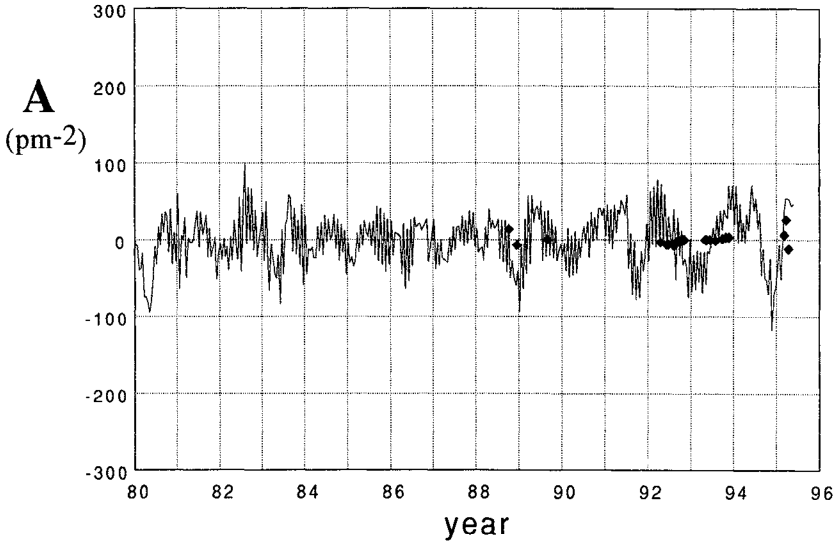

Figure 3. The sine coefficient A as solved for from laser ranging data, assuming the once-per-revolution force is entirely along track. This curve is not analyzed in this paper; it is shown only for completeness. Here pm is picometer (10^{-12} m).

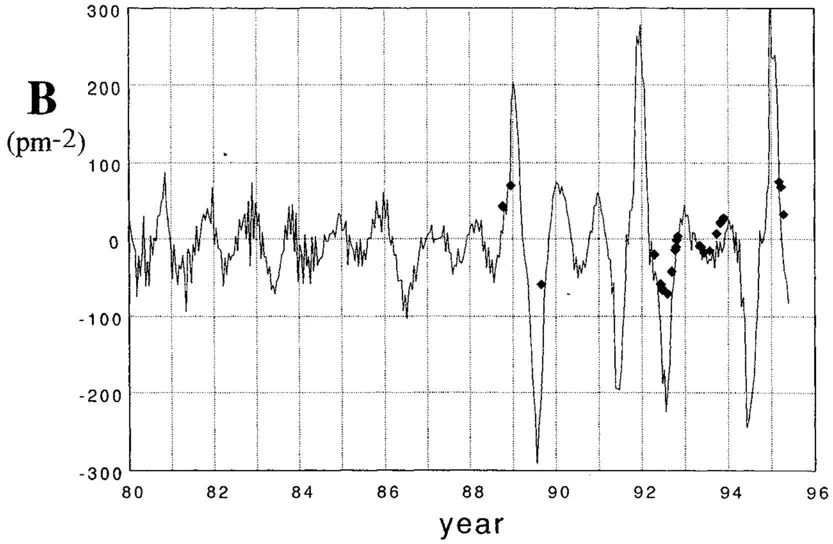

Figure 4. The solid line is the cosine coefficient *B* as solved for from laser ranging data assuming the once-per-revolution force is entirely along track. The diamonds give *B* for the spin axis positions recovered from Sun glint data, after calibrating the amplitude from a least squares fit of the early data when the spin axis remains nearly fixed in space. There are 27 determinations of the spin axis position, but some points are so close together they cannot be distinguished. Here pm is picometer (10^{-12} m).

[1991] and *Sengoku et al.* [1995], although it should certainly be included in future research. Data prior to 1980 is not used due to the noisy quality of the lasers at that time.

Analysis of Data

The spin axis vector at orbit injection was $\theta = 158°$ and $\lambda = 133°$. When this position is substituted in the above equations, the last two terms in the *B* coefficient dominate and *B* is almost purely sinusoidal with a period of 1 year. This is in qualitative agreement with the first few years of data shown in Figure 4. An analysis of the thermal drag signature in the along-track acceleration data indicates that the spin vector did not move significantly from the launch orientation from 1980 to 1983 [*Rubincam*, 1990b]. These conclusions, derived from the SLR data, agree with the presumption that the satellite was spinning too fast to precess very far from its position at launch; the gyroscopic inertia was too big. This view is confirmed by the analytic models of the dynamics of the spin axis [*Bertotti and Iess*, 1991; *Habib et al.*, 1994; *Farinella et al.*, 1996]. Thus from the thermal drag analysis *B* is expected to be largely sinusoidal during these years, in agreement with Figure 4.

A least squares fit of a sinusoid to the 1980-1983 data assuming that the spin axis stayed at its initial position at orbit injection gives an amplitude of 38 ± 4 x 10^{-12} m s^{-2} and a phase lag of about 204°± 7° relative to the beginning of calendar year 1980 (see Figure 5). The expected phase lag from the last two terms in *B* is 197°, so that the phase from theory

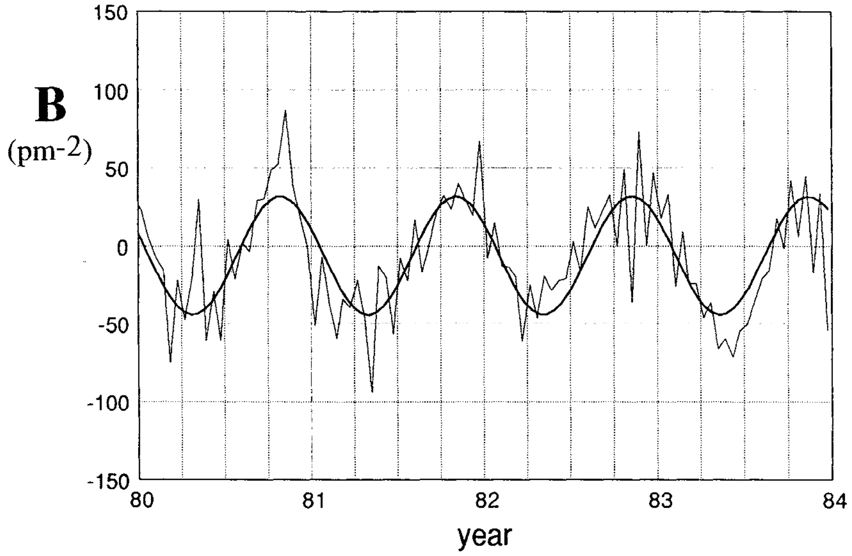

Figure 5. The least squares sinusoidal fit of the 1980-1983 data shown in Figure 4. Here pm is picometer (10^{-12} m).

gives very good agreement with the data, indicating that most of the signal appears to be due to solar heating as proposed here. There is also a small constant term of $-6 \pm 3 \times 10^{-12}$ m s^{-2} in B, which may indicate a small unmodeled force, although the uncertainty is big.

Thus having confidence that the thermal force interpretation is largely correct, the amplitude of the sinusoid for 1980-1983 can be used to calibrate the thermal force constant b. It turns out to have a value of $b = 155 \times 10^{-12}$ m s^{-2}. We use this value in turn to compute B for the 27 spin axis positions listed in Table 1 obtained from the Sun glint data [*Currie*, 1994; *Currie et al.*, 1995, 1996]. In addition to published data these positions also include data that will appear in the doctoral dissertation of P. Avizonis. The diamonds in 1992-1995 shown in Figure 4 follow the general shape of the curve, most particularly the trough in 1992. Unfortunately, there are no data points between the June and July values at the bottom of the trough.

Discussion

Figure 4 illustrates the values of B derived from the SLR data. The solar heating force clearly has the right signature. However, the values of B obtained from the use of the spin axis observations (diamonds) on either side of the 1992 trough underestimates the strength of the signal. Due to the smooth dependence of the relations in (5), an interpolation between the June and July points indicates that the trough will get only 2% deeper than shown by the June and July positions. Moreover, it is clear from (2) that the maximum and minimum values that B can achieve are $\pm b$. Since we recover a value of $b = 155 \times 10^{-12}$ m s^{-2}, it appears that our model cannot explain the extreme peaks and troughs, even if the spin axis underwent large excursions (which are not observed in the data). This indicates that our model is too simple or that other forces are at work, or both.

Table 1. Spin axis positions for LAGEOS I

Date	Year	θ (deg)	λ (deg)
Sept. 30	1988	163	152
Dec. 9	1988	173	305
Aug. 23	1989	168	345
April 6	1992	166	274
May 30	1992	168	301
June 2	1992	167	305
June 10	1992	166	308
June 13	1992	167	309
July 29	1992	171	314
Sept. 1	1992	171	0
Sept. 29	1992	170	21
Oct. 2	1992	170	25
Oct. 7	1992	169	28
Oct. 15	1992	170	30
Oct. 23	1992	169	39
April 24	1993	164	88
April 28	1993	163	89
May 7	1993	161	94
June 2	1993	164	90
July 17	1993	163	93
Sept. 15	1993	162	101
Oct. 16	1993	165	108
Nov. 11	1993	166	115
Nov. 13	1993	168	116
Feb. 25	1995	156	301
March 13	1995	157	358
April 5	1995	160	37

Recovered from Sun glint data. Here θ is the colatitude and λ is the east longitude of the satellite spin axis in the xyz system shown in Figure 2.

Why might our model be too simple? It could be that the solar heating varies more strongly than ($\hat{s}\cdot\hat{r}_S$). There may be several reasons for the stronger dependence. One is poor thermal coupling between the hemispheres. The satellite is not solid; the northern and southern hemispheres are bolted together. Perhaps b has a temperature dependence. A second possible reason is the four germanium retroreflectors on the satellite. When the temperature of these retroreflectors gets too high, they undergo a kind of runaway greenhouse effect and become very hot [*Gaposchkin and Pearlman*, 1985]. A high Sun angle and the present slow rotation rate (period ≈ 200 s) of LAGEOS I may be causing this to happen, creating unmodeled "hot spots." A third possible reason is albedo variations on LAGEOS, with the Sun "seeing" different areas of light and dark on the satellite at different Sun angles.

In favor of a stronger solar heating dependence than that given by our model is that the Sun was shining at a maximum angle of about 16° with respect to LAGEOS's equator during the 1980-1983 time frame, but was shining more strongly on LAGEOS's southern hemisphere at an angle of about 30° at the time of the 1992 trough. However, more research is needed to determine which, if any, of our three explanations proposed above is correct.

In the above only the along-track component of the solar heating force was used to explain the data, since the radial acceleration is too highly correlated with the along-track acceleration to solve for independently in the GEODYN orbit determination program (D. Rowlands, private communication, 1996). But a theoretical analysis of the eccentricity excitation [*Tapley et al.*, 1993] using both the radial and the along-track components of the solar once-per-revolution force in Lagrange's equations indicates that the only error in the foregoing is to overestimate b by a factor of 3/2. Hence a naive multiplication of our least squares value by 2/3 gives $b = 103 \times 10^{-12}$ m s^{-2}, in good agreement with the dominant term in Slabinski's thermal model, which gives 102.5×10^{-12} m s^{-2} (V. J. Slabinski, submitted manuscript, 1995). However, our thermal model may be too simple, as stated above.

A value of $b = 103 \times 10^{-12}$ m s^{-2} corresponds to a temperature difference between the north and south poles of LAGEOS I of roughly 27 K, assuming the Sun stands over either pole. To get the temperature difference at other orientations, this figure must be multiplied by the cosine of the angle between the spin axis and the line joining the satellite to the Sun.

While we do not use the sine part of the force to solve for b, we do show our model values based on the spin axis orientation (Figure 3). Our values do not agree particularly well with the SLR data, but the sine component appears to be afflicted with more noise (or perhaps unmodeled forces) than the cosine component.

Other possible explanations for our failure to explain the extreme peaks and troughs of Figure 4 is that other forces, such as the anisotropic reflection force [*Rubincam et al.*, 1987, p. 11,667; *Scharroo et al.*, 1991; *Sengoku et al.*, 1995] and atmospheric tides [*Nerem et al.*, 1994] are at work. Although our results here tend to argue against the tidal explanation, perhaps some of the remaining signal after the thermal force is taken out is due to tides.

Summary

In summary, we find that photon thrust due to solar heating appears to explain much of the unknown forces acting on LAGEOS I (Figure 4). Solar heating seems to have the right signature, and the recovered amplitude of the thermal force from the 1980-1983 data agrees well with Slabinski's thermal model. However, a better thermal model may be needed to account for the highest peaks and deepest troughs of Figure 4. This force will of course also be present on LAGEOS II. Other forces, such as the anisotropic reflection force, should also be considered. The observed forces may not have an atmospheric tidal origin, as proposed by *Nerem et al.* [1994].

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